UKRI Verifiability Node

Power Dynamics in Resource-Exchange Graph Grammars

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TermGraph Workshop
April 7, 2024

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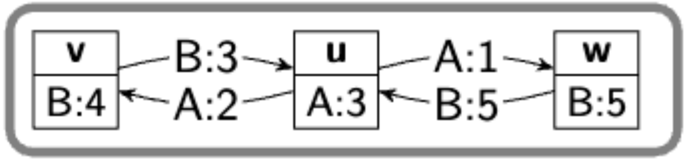
Motivation

- Analysis of emergent behaviours in complex systems:
  - Particularly, dynamic topologies and group formation
- Why Graph Grammars (GGs)?
  - Dynamic:
    - Network dynamics naturally captured using graph rewriting rules
  - Multiscale:
    - Enable capturing dynamics at multiple scales in a unified model
  - Modular:
    - Combination of different aspects can be modelled incrementally
  - Rigorous:
    - Rich underlying theory enables optimisation while preserving correctness

Case Study

Each agent in a network:

- Has a set of resources
- Decides how much of which resource to give to its contacts
- Has a subjective value for each resource
- Subscribes to a Relational Model (RM) [1]:
  - Altruism
  - Reciprocity
  - Opportunism
  - Status



vals	A	B
v	1	2
u	2	1
w	2	2

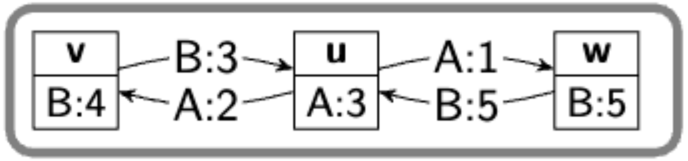
Power according to [2]:

- **Inverse of dependence**

[1] Fiske, A. P. (1992). The four elementary forms of sociality: framework for a unified theory of social relations. *Psychological review*, *99*(4), 689.
[2] Cook, K. S., Emerson, R. M., Gillmore, M. R., & Yamagishi, T. (1983). The distribution of power in exchange networks: Theory and experimental results. *American journal of sociology*, *89*(2), 275-305.

Example: Power based on Dependency

- **Dependency** of u on w:
  - How much B is given to u by w relative to max amount of B given to u by v
  - How much u values B
- **Power** of w over u:
  - Dependency of u on w

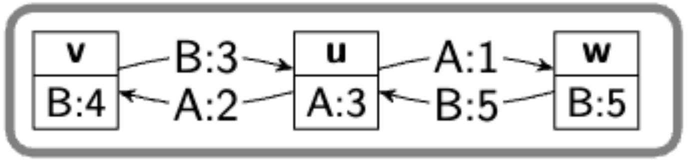


vals	A	B
v	1	2
u	2	1
w	2	2

Dep(v, u) = (2 – 0) x val(v, A) = 2
Dep(u, v) = (3 – 5) x val(u, B) ~ 0
Pow(u, v) = 2   Pow(v, u) = 0

Dep(w, u) = (1 – 0) x val(w, A) = 2
Dep(u, w) = (5 – 3) x val(u, B) = 2
Pow(u, w) = 0   Pow(w, u) = 0

Example: Actual vs Subjective Power



vals	A	B
v	1	2
u	2	1
w	2	2

Actual Power:

- Value of resources from perspective of recipients

  e.g. Dep(u, w) is based on **val(u, B)**

Dep(w, u) = (1 – 0) x val(w, A) = 2
Dep(u, w) = (5 – 3) x **val(u, B)** = 2

Pow(u, w) = 2 – 2 = 0
Pow(w, u) = 2 – 2 = 0

Used for determining **balanced networks**.

Subjective Power:

- Value of resources from perspective of a specific agent

  e.g. Sub_Dep(u, w, w) is based on **val(w, B)**

Sub_Dep(w, u, w) = (1 – 0) x val(w, A) = 2
Sub_Dep(u, w, w) = (5 – 3) x **val(w, B)** = 4

Sub_Pow(w, u, w) = 4 – 2 = 2
Sub_Pow(u, w, w) = 2 – 2 ~ 0

Used for **RM constraint solving**.

Power Dependence Theory (PDT)

- Power imbalance:
  - Causes instability
  - Triggers balancing operations:

Balancing Approach	Motivational	Structural
Decreasing Dep of Self	Withdrawal	Network Extension
Increasing Dep of Other	Investing More	Coalition Formation

$$dep(x,y,r) = \begin{cases} (flow(y,x,r) - \max\limits_{k \neq y} flow(k,x,r)) \times val(x,r) & flow(y,x,r) > \max\limits_{k \neq y} flow(k,x,r) \\ 0 & \text{o.w.} \end{cases}$$

[1] Cook, K. S., Emerson, R. M., Gillmore, M. R., & Yamagishi, T. (1983). The distribution of power in exchange networks: Theory and experimental results. *American journal of sociology*, *89*(2), 275-305.

Relational Models Theory (RMT)

- Four elementary Relational Models (RMs) provide a comprehensive basis for all social life

Relational Model	**RM**	**Motivation**	**Description**
Communal Sharing	CS	Altruism	All contacts receive an **equal** amount
Equality Matching	EM	Reciprocity	For each contact, weighted amounts received **equals** weighted amounts sent
Market Pricing	MP	Opportunism	For each contact, weighted amounts received **exceeds** weighted amounts sent
Authority Ranking	AR	Status	For each contact v, weighted amount sent to v **exceeds** weighted amount sent to v by any others

[1] Fiske, A. P. (1992). The four elementary forms of sociality: framework for a unified theory of social relations. *Psychological review*, *99*(4), 689.

Research Questions

- If we increase the amount of resource for an agent, what would be the impact on:
  - subjective power
  - RMs
- Variations to explore:
  1. **Benchmark**:
     - Subjective values can vary
  2. **Impartial**:
     - Every agent is impartial towards resources
  3. **Consensus**:
     - Global values for resources

Methodology

Unfolding theory [1]:

Given a finite graph grammar derives all reachable graphs in the most efficient construction

Extended unfolding theory supports [2]:

Negative Application Conditions (NACs)

Symbolic data attributes

Lazy approach:

Purely structural unfolding

Constraint solving on attributes using an SMT-solver

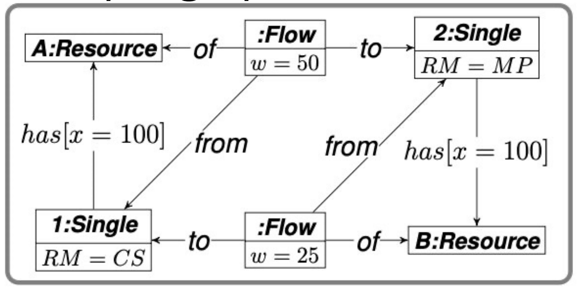
[1] Baldan, P. (2000). *Modelling concurrent computations: from contextual Petri nets to graph grammars* (Doctoral dissertation, PhD thesis, Department of Computer Science, University of Pisa, 2000. Available as technical report n. TD-1/00).

[2] Saadat, M. G. (2022). *Applications of category theory in analysis of complex systems* (Doctoral dissertation, University of Leicester).

Graph Transformation System

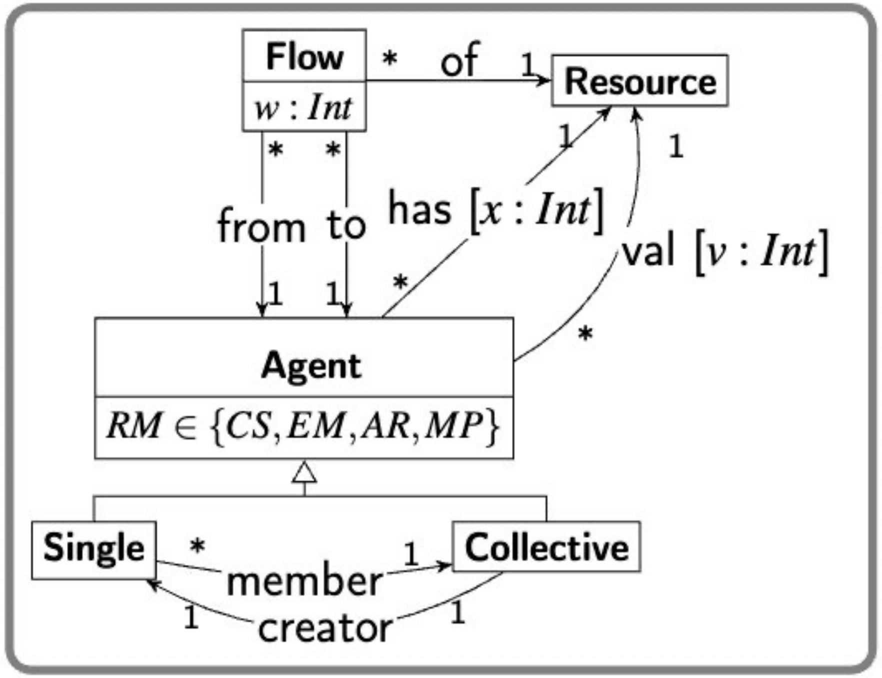
Nodes:
- Agent [attribute: RM]:
  - Single
  - Collective
- Flow [attribute: w]
- Resource

Sample graph:



Arcs:
- member:
  - from Single to Collective
- creator:
  - from Collective to Single
- from:
  - from Agent to Flow
- to:
  - from Flow to Agent
- of:
  - from Glow to Resource
- has [attribute: x]:
  - from Agent to Resource
- val [attribute: v]:
  - from Agent to Resource

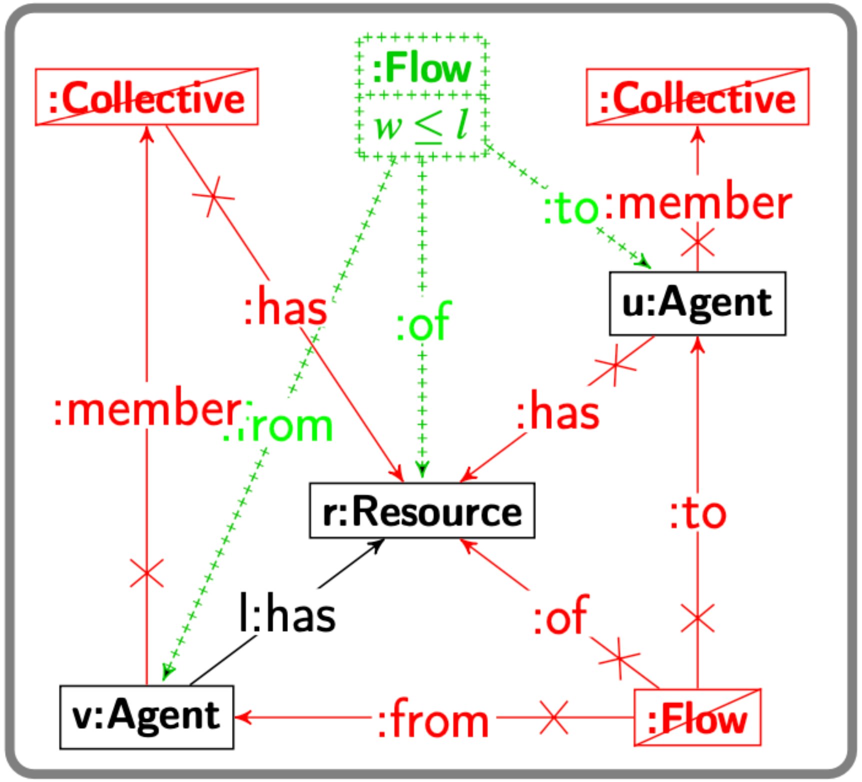
Type graph:



Create flow rule:

- If:
  - No resource *r* flows from agent *v* to agent *u*

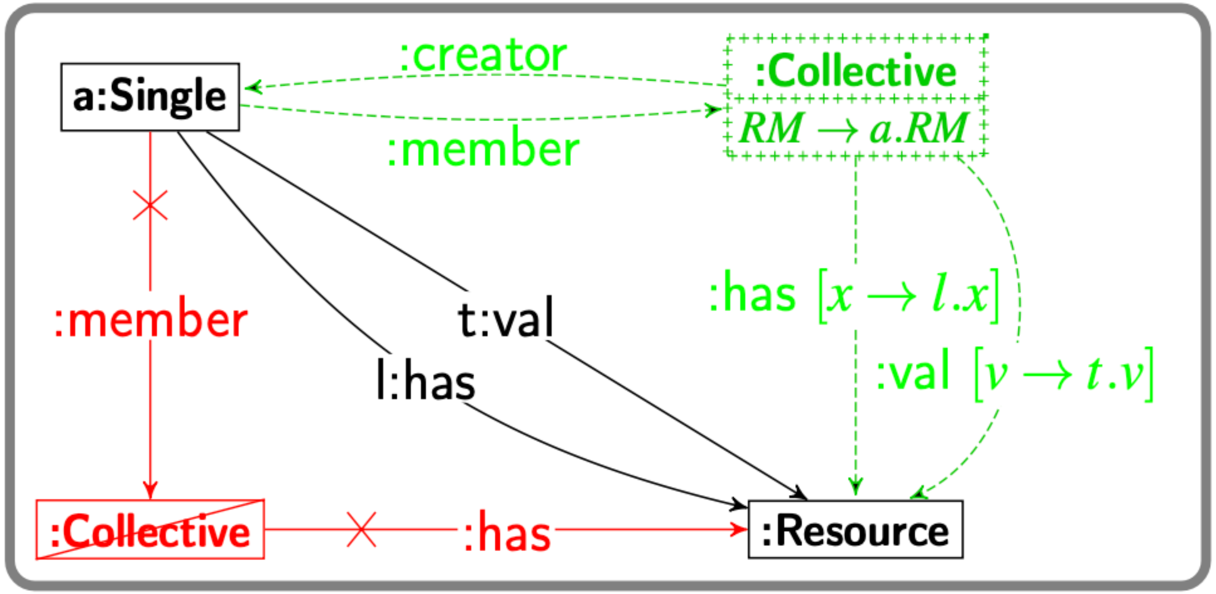
  and
  - Neither *v* nor *u* is a member of an *r*-Collective
- Then:
  - A flow can be created up to the amount of *r* that agent *v* possesses



Create collective rule:

- If:
  - A single agent a has resource r

  and
  - Agent a is not a member of an *r*-Collective
- Then:
  - Agent a can create an r-Collective
- The new collective:
  - Inherits the initial amount of r from a
  - Adopts a's subjective resource values

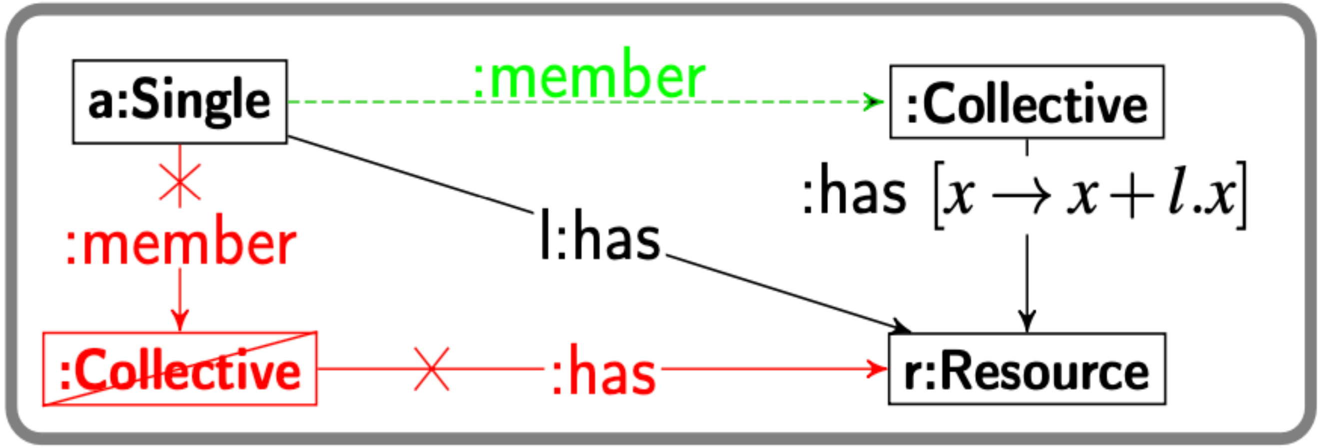


Join collective rule:

- If:
  - A single agent a has resource r

  and

  - Agent a is not a member of an *r*-Collective
- Then:
  - Agent a can join an r-Collective
- The new collective:
  - Inherits the initial amount of r from a

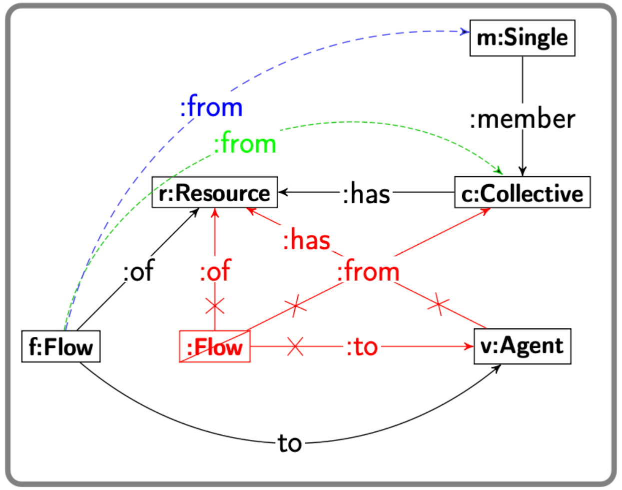


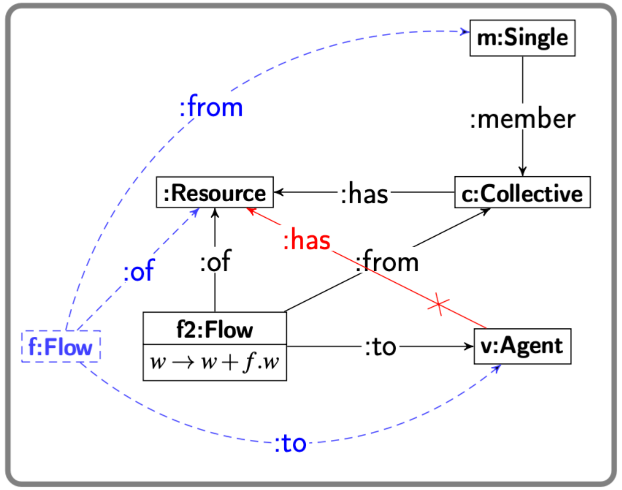
Redirect Flows rules:

- If:
  - There is a flow f of r from a member m of an r-Collective c to agent v

  and

  - Agent v does not have resource r
- Then:
  - If there is no flow of r from c to v,
    - then m's role shifts to c
  - If there is a flow f2 of r from c to v,
    - then f is deleted and its amount is added to f2



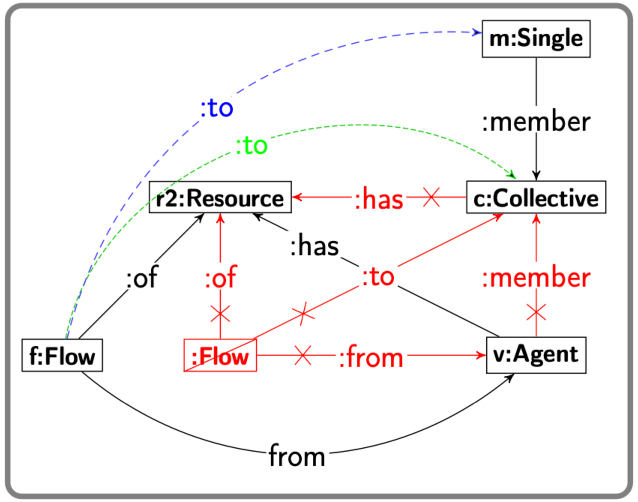


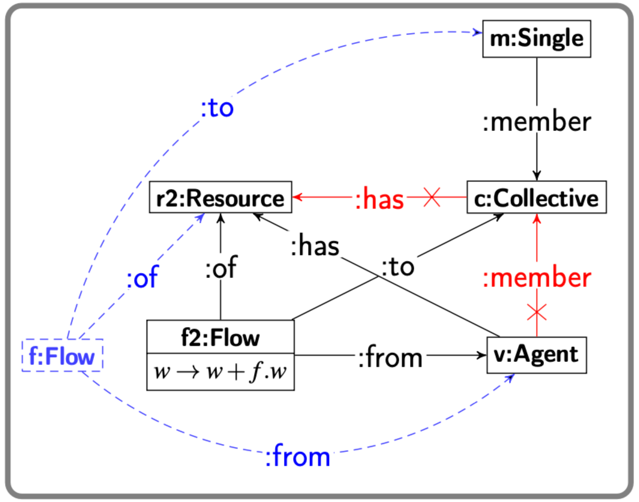
Redirect Flows rules:

- If:
  - There is a flow f of r **to** a member m of an r-Collective c **from** agent v

  and

  - Agent v has resource r
- Then:
  - If there is no flow of r2 from v to c,
    - then m's role shifts to c
  - If there is a flow f2 of r2 from v to c,
    - then f is deleted and its amount is added to f2





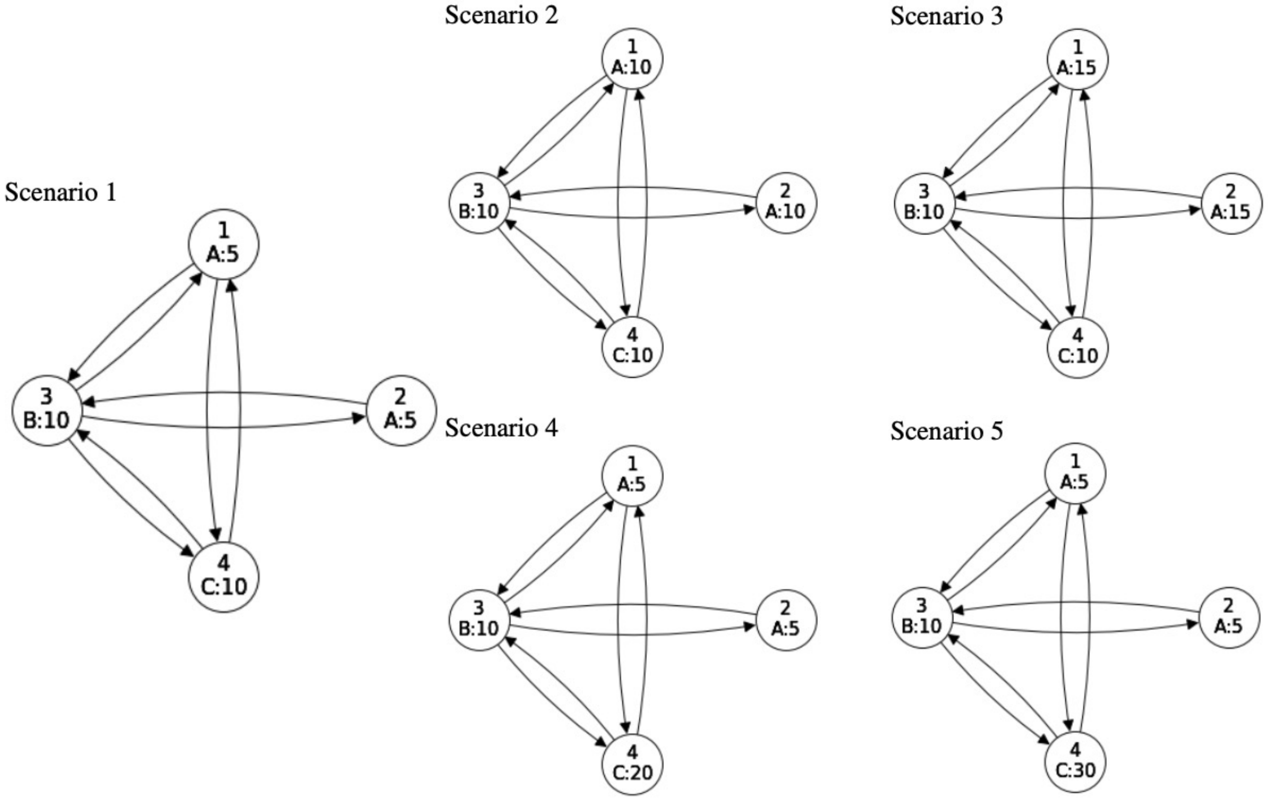
Graph Grammar – Start Graphs

Goal:

Compare the effect of disparities in resource allocation on subjective power and RMs in two cases:
- Rich agent is a **Single**
- Rich agent is a **Collective**

Scenarios:

1. Benchmark
2. Agents 1 & 2 has **twice** as much
3. Agents 1 & 2 has **three times** as much
4. Agent 4 has **twice** as much
5. Agent 4 has **three times** as much



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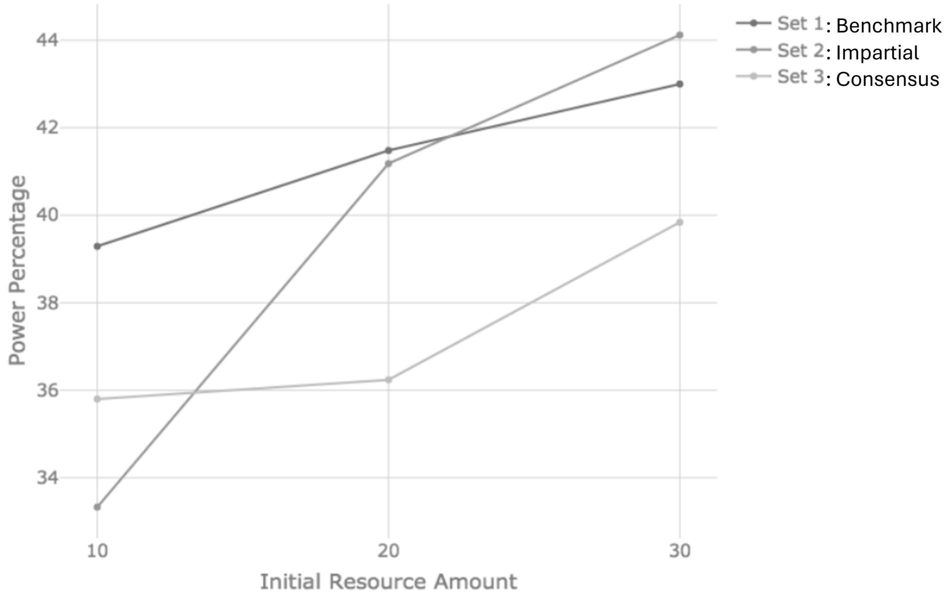
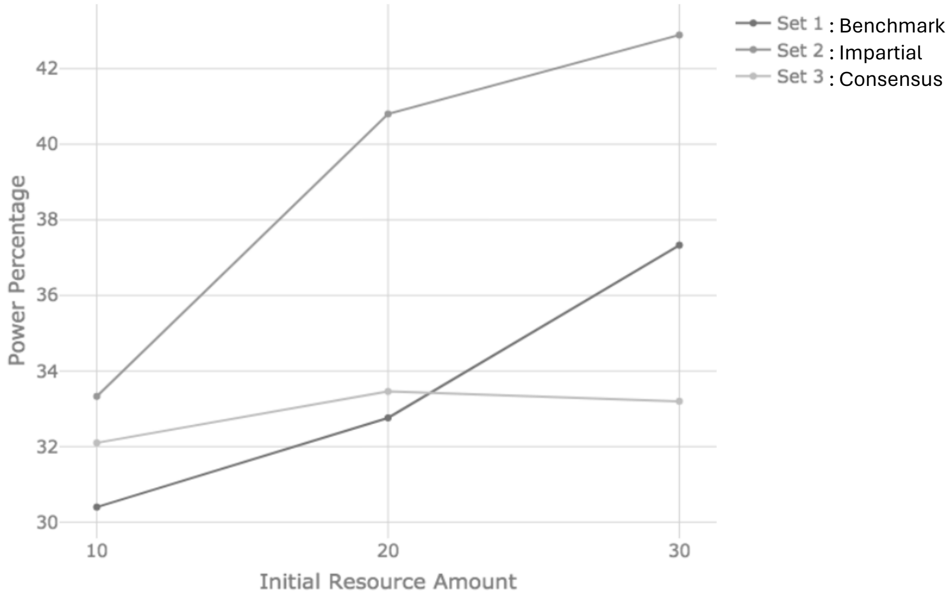
Results – Average Subjective Power: Comparison of Conditions



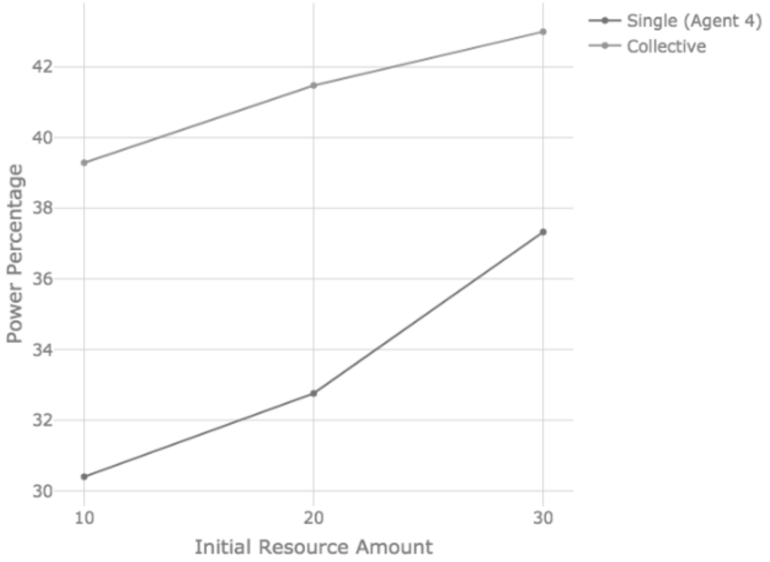
Relative Power of Rich Collective



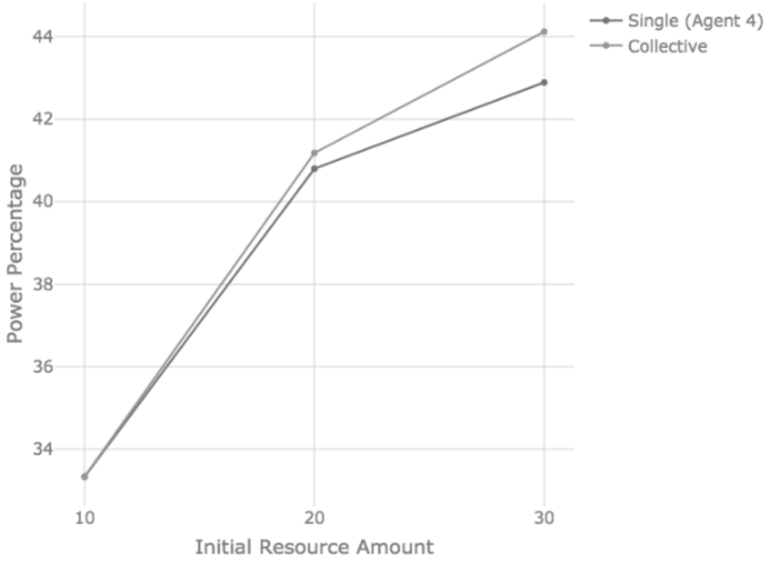
Relative Power of Rich Single

- Consensus condition (light grey) is most stable under perturbation of initial resource amounts

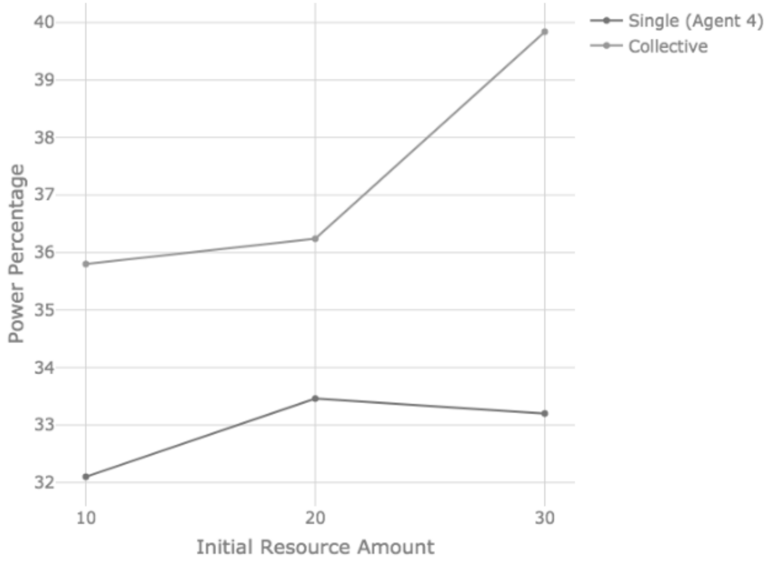
Results – Average Subjective Power : Single vs Collective Agent



Set 1: Benchmark



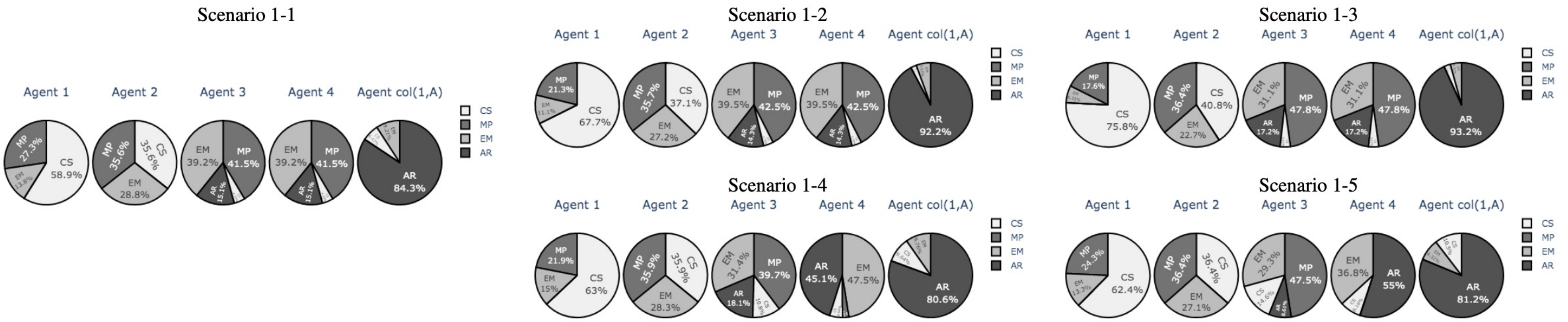
Set 2: Impartial



Set 3: Consensus

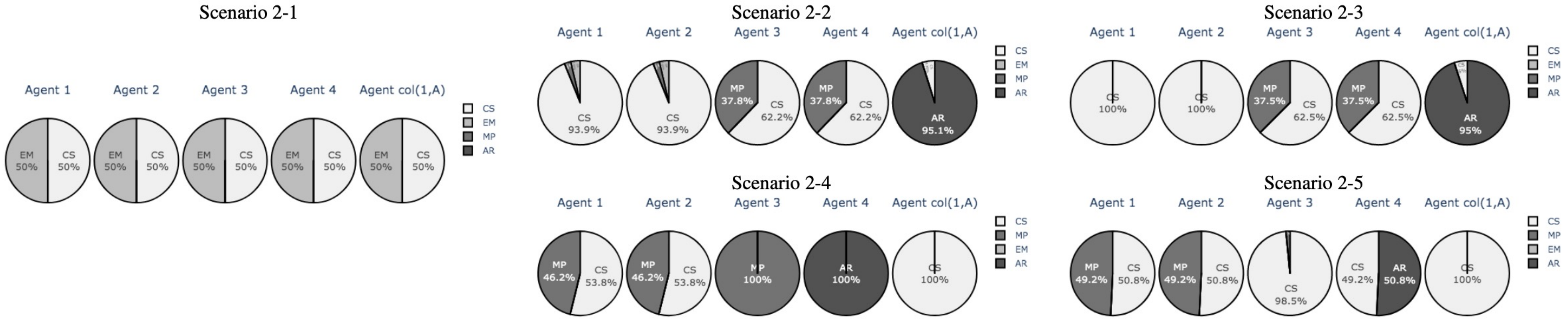
- A rich Collective consistently gains more power than a rich Single with the same initial boost
- Under Impartial condition, power gain diminishes as resource amount increases

Results – Average RMs: Benchmark Condition



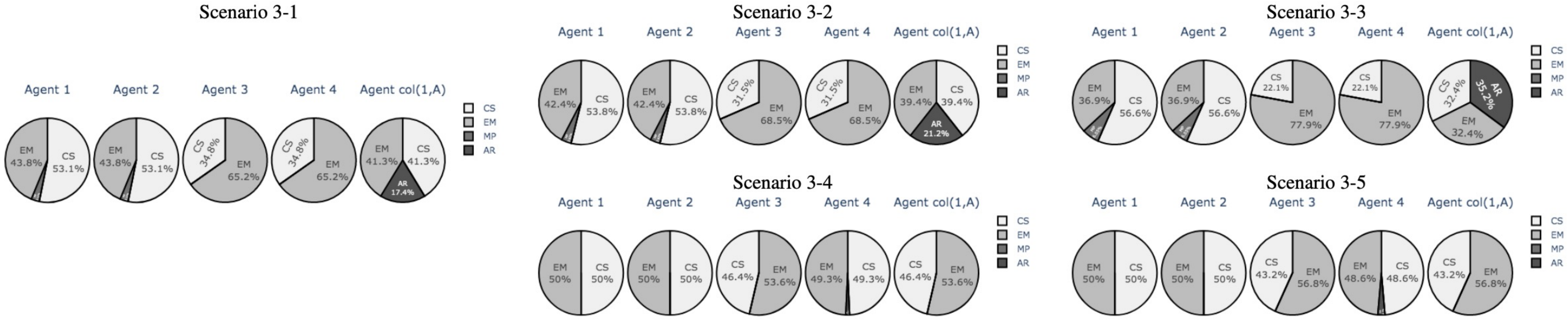
- In absence of inequality, Collective dominates the network
- Rich Collective dominates the system slightly more, increasing altruism inwardly
- Rich Single doesn't have a significant impact on RMs

Results – Average RMs: Impartial Condition



- In absence of inequality, a utopian state
- Rich Collective becomes outwardly authoritarian (inwardly altruistic)
- Rich Single stratifies roles, leads to outwardly altruistic Collective (inwardly more opportunistic than before)

Results – Average RMs: Consensus Condition



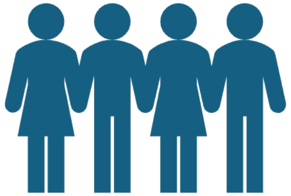
- In absence of inequality, Collective is slightly dominant
- Rich Collective becomes more authoritarian
- Rich Single leads to a stable quasi-utopian state

Case Study – Emergent Behaviours

A rational basis for socio-economic clustering:

Homogeneous values (consensus) lead to stable altruistic and reciprocal attitudes

Diverse values (benchmark and impartial) lead to stratification of roles, often variable depending on context



Investing in Single vs Collective agents:

Rich Single can lead to more altruistic and reciprocal attitudes

Rich Collective consistently dominates the system

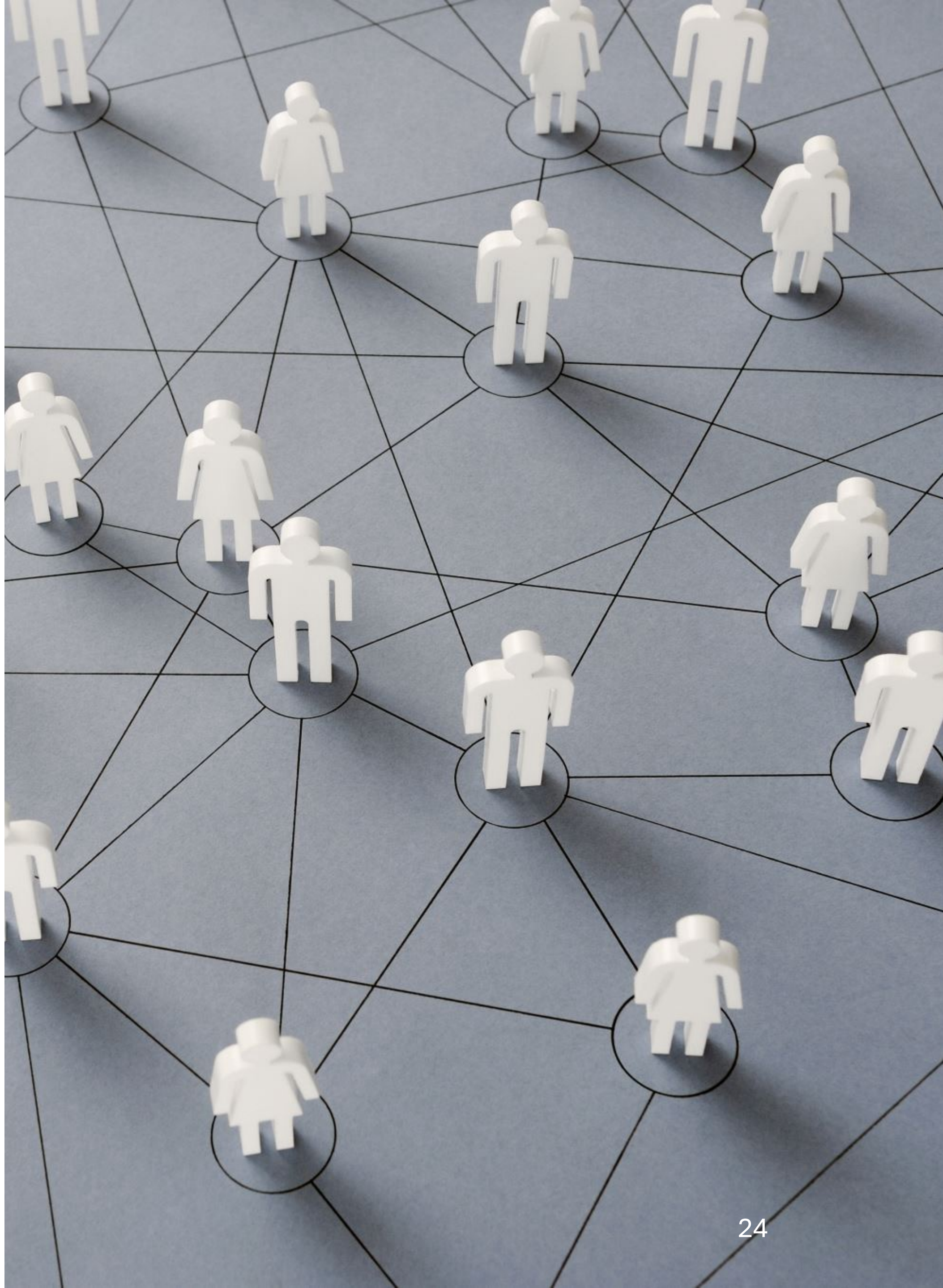
Limitations of Study

This work aimed to explore emergent behaviours resulting from the PDT and RMT in resource-exchange networks

Validating the PDT and RMT against the reality falls out of the scope of this project

Concluding Remarks

- Despite modest size of example:
  - observed interesting emergent behaviours and possible causal explanations
- Despite efficiency challenges:
  - unfolding theory offers promise due to its rich theoretical foundations
- Potential future work:
  - Generic implementation of extended theory of unfolding
  - Application in scaled-down models of real-world scenarios to:
    - identify resource related conflicts
    - provide insights as to how they can be managed/resolved
  - Integration with ML algorithms to enable predictive forecasts for:
    - Power distribution
    - Organisational models
    - Government types
    - Cultural norms



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Thank you