ZX-calculus as a graphical rewriting language for quantum computing

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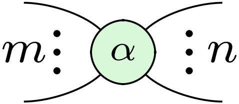
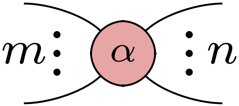
April 7, 2024

ZX-calculus

Spiders

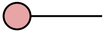


$$\xmapsto{[\![\cdot]\!]} \quad |0\rangle^{\otimes n} \langle 0|^{\otimes m} + e^{i\alpha} |1\rangle^{\otimes n} \langle 1|^{\otimes m}$$

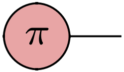


$$\xmapsto{[\![\cdot]\!]} \quad |+\rangle^{\otimes n} \langle +|^{\otimes m} + e^{i\alpha} |-\rangle^{\otimes n} \langle -|^{\otimes m}$$

Computational basis states

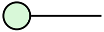


$$\xmapsto{[\![\cdot]\!]} \quad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

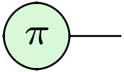


$$\xmapsto{[\![\cdot]\!]} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Superposition

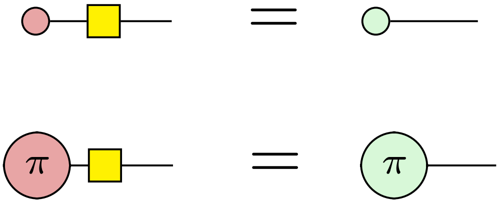


$$\xmapsto{[\![\cdot]\!]} \quad |+\rangle \quad = \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

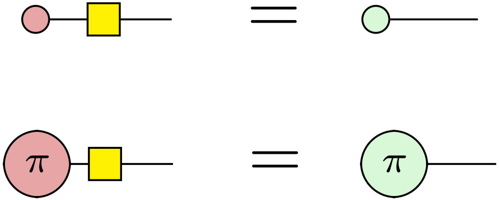


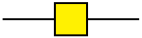
$$\xmapsto{[\![\cdot]\!]} \quad |-\rangle \quad = \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

Hadamard box



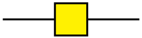
Hadamard box





$$\stackrel{[\![\cdot]\!]}{\longmapsto} \quad |+\rangle \langle 0| \; + \; |-\rangle \langle 1|$$

Hadamard matrix

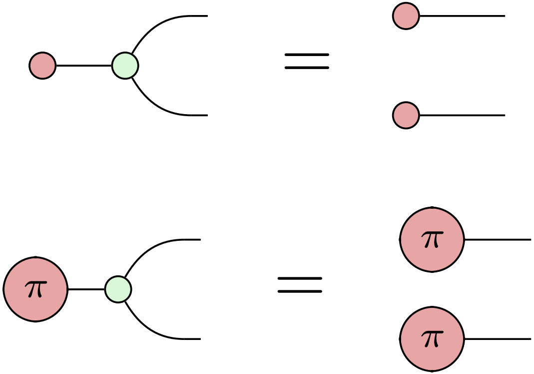


$$\xrightarrow{[\![\cdot]\!]} \quad |+\rangle\langle 0| + |-\rangle\langle 1|$$

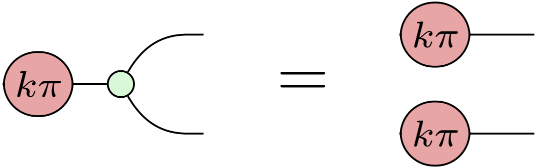
$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 1 & 0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1\\ 0 & -1\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

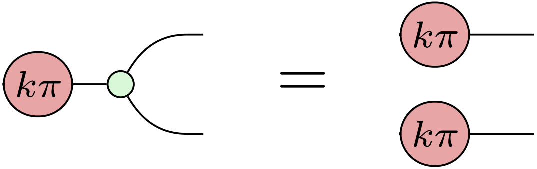
Z-spider

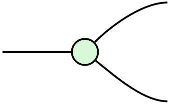


Z-spider



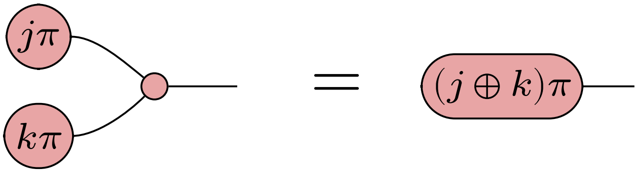
Z-spider



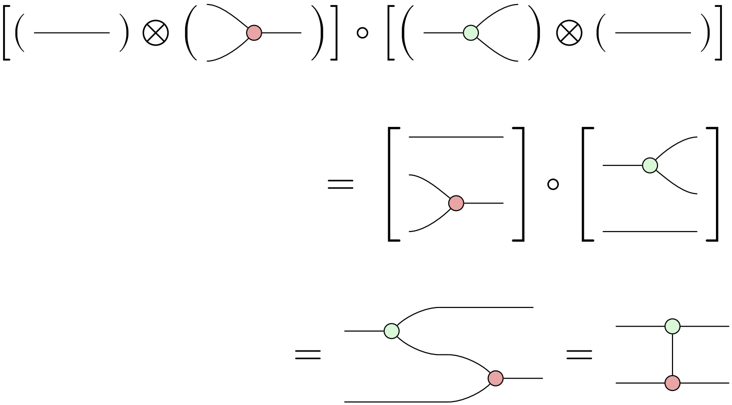


$$\xmapsto{[\![\cdot]\!]} \quad |0\rangle \langle 0, 0| + |1\rangle \langle 1, 1|$$

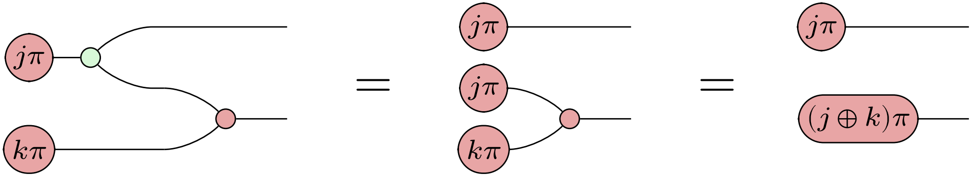
X-spider



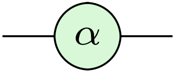
Composition



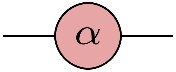
CNOT gate



Phases

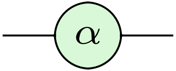


$$= \quad |0\rangle\langle 0| + e^{i\alpha}|1\rangle\langle 1|$$

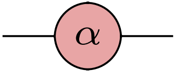


$$= \quad |+\rangle\langle +| + e^{i\alpha}|-\rangle\langle -|$$

Phases

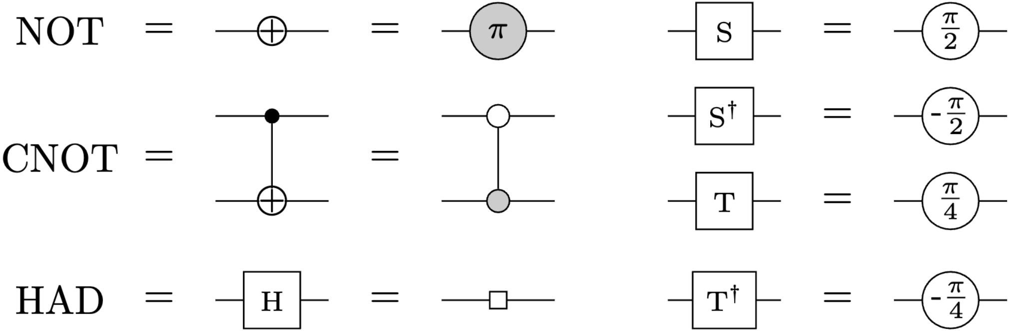


$$= \begin{bmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{bmatrix}$$

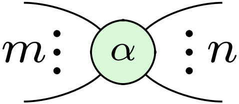


$$= \frac{1}{2}\begin{bmatrix} 1 + e^{i\alpha} & 1 - e^{i\alpha} \\ 1 - e^{i\alpha} & e^{i\alpha} \end{bmatrix}$$

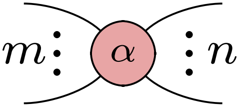
Quantum gates



Spiders



$$\overset{[\![\cdot]\!]}{\longmapsto} \quad |0\rangle^{\otimes n}\langle 0|^{\otimes m} + e^{i\alpha}|1\rangle^{\otimes n}\langle 1|^{\otimes m}$$



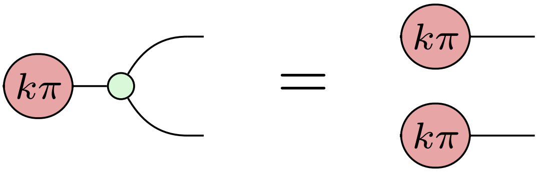
$$\overset{[\![\cdot]\!]}{\longmapsto} \quad |+\rangle^{\otimes n}\langle +|^{\otimes m} + e^{i\alpha}|-\rangle^{\otimes n}\langle -|^{\otimes m}$$

Theorem (Universality)

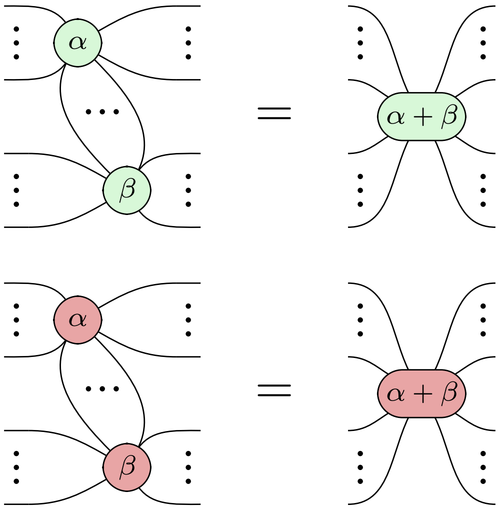
Any linear map between qubits can be expressed in terms of ZX diagrams.

Rewrite rules

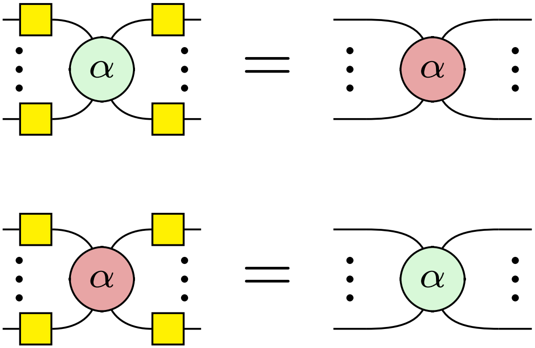
Copy



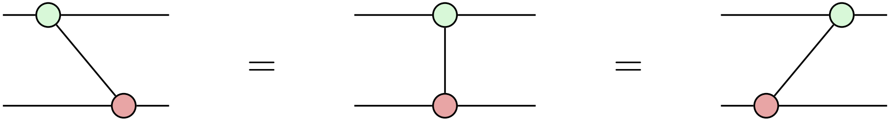
Fusion



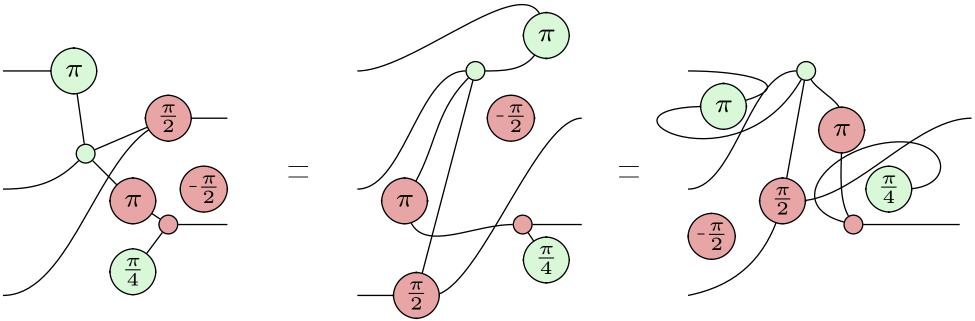
Color



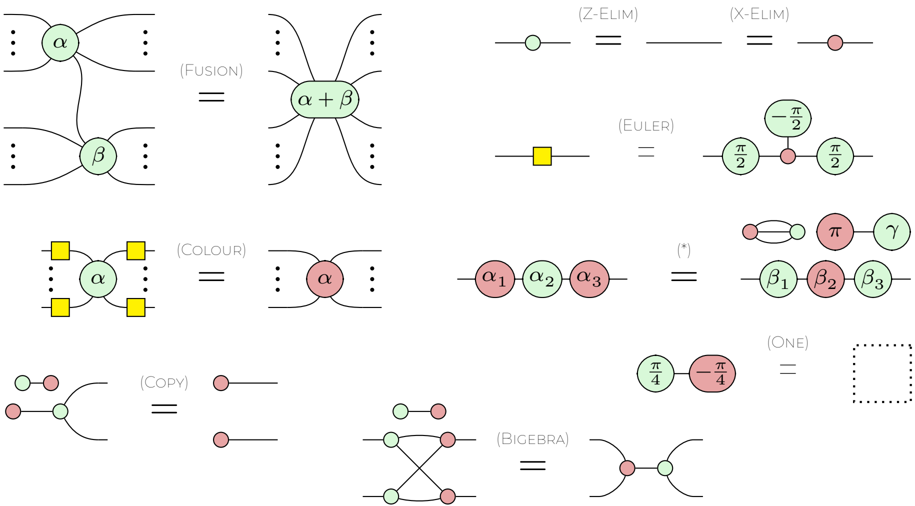
Only Connectivity Matters



Only Connectivity Matters



Axioms



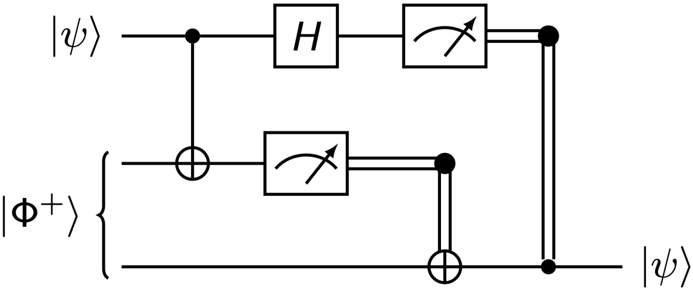
Completeness

Theorem (Completeness)

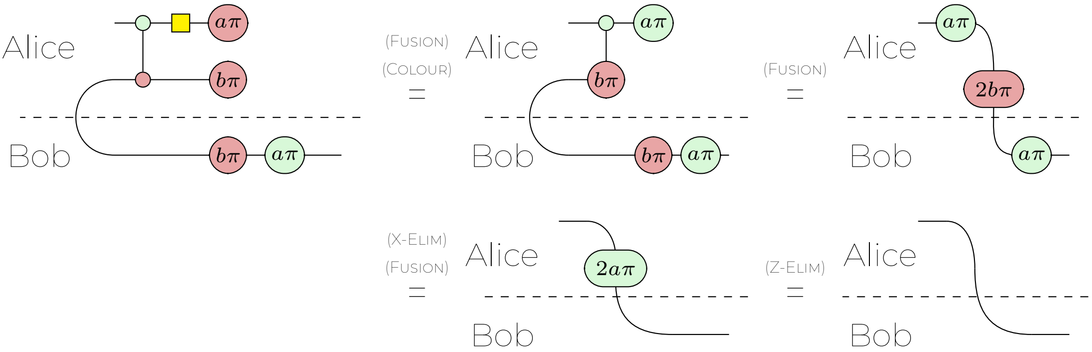
Any equation that holds for linear maps between qubits can be derived in ZX-calculus.

Examples

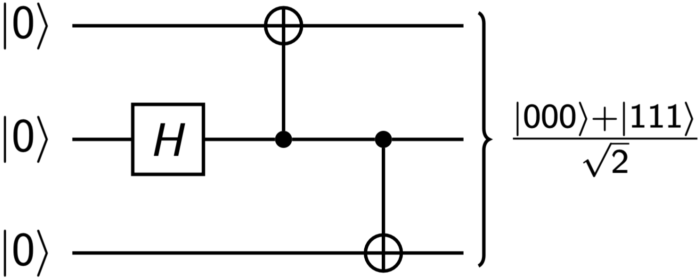
Quantum Teleportation



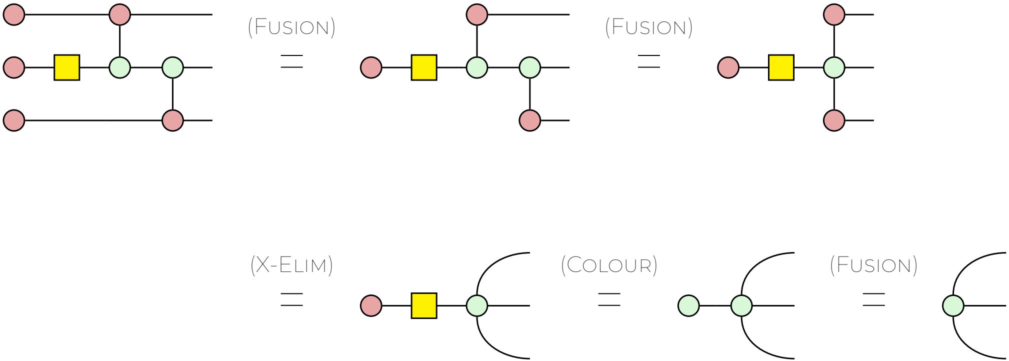
Quantum Teleportation



GHZ state preparation



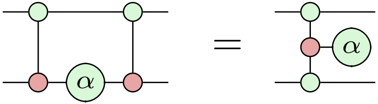
GHZ state preparation



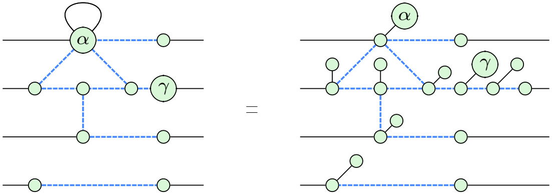
Extensions

Applications: ZX-calculus

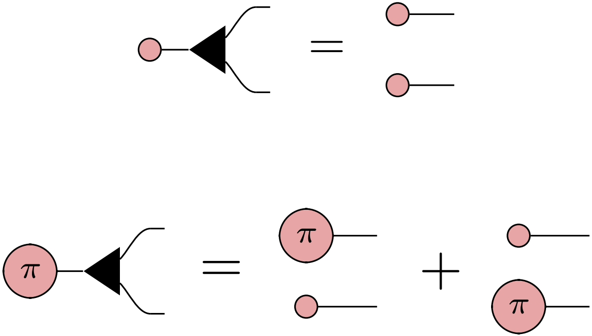
Quantum Circuit Optimisation



Measurement-Based Quantum Computing

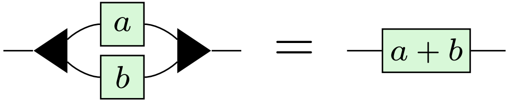


W node

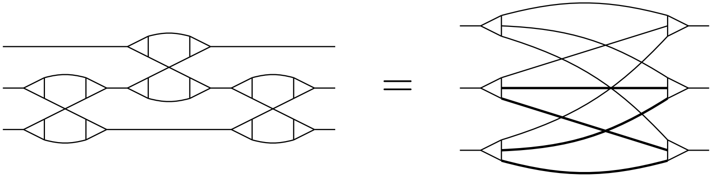


ZW-calculus

Summation



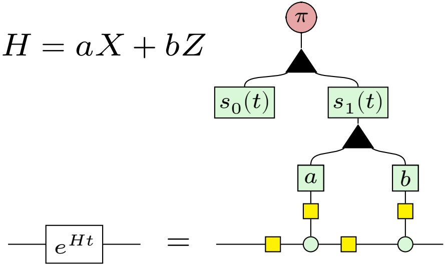
Linear Optical Quantum Computing



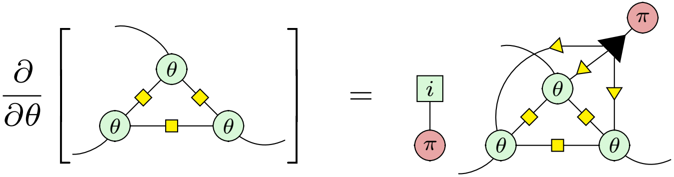
ZXW-calculus

Hamiltonians

$$H = aX + bZ$$



Differentiation and integration



Thank you!

Overview

1. Introduction
2. Generators
3. Rewrite rules
4. Examples
5. Extensions