Transformation of DPO Grammars into Hypergraph Lambek Grammars With The Conjunctive Kleene Star

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- Another approach is categorial grammars.
- One assigns categories (=types) to symbols of an alphabet.
- A general mechanism deals with sequences of categories.
- A string is correct if one can replace each its symbol by a corresponding category and obtain a sequence of categories, which is accepted by a general mechanism.

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- Example: $p/(q\backslash p)$, $(p \cdot q)/(q \cdot p)$ are types.
- A sequent is a structure of the form $A_1, \ldots, A_n \to B$ where A_i, B are types.

Lambek calculus: Axiom and Rules

$$\frac{}{A \to A}\ (\mathrm{Ax})$$

$$\frac{\Pi \to A \quad \Gamma, B, \Delta \to C}{\Gamma, \Pi, A \backslash B, \Delta \to C}\ (\backslash \to) \qquad \frac{A, \Pi \to B}{\Pi \to A \backslash B}\ (\to \backslash)$$

$$\frac{\Pi \to A \quad \Gamma, B, \Delta \to C}{\Gamma, B / A, \Pi, \Delta \to C}\ (/ \to) \qquad \frac{\Pi, A \to B}{\Pi \to B / A}\ (\to /)$$

$$\frac{\Gamma, A, B, \Delta \to C}{\Gamma, A \cdot B, \Delta \to C}\ (\cdot \to) \qquad \frac{\Pi \to A \quad \Psi \to B}{\Pi, \Psi \to A \cdot B}\ (\to \cdot)$$

$\Gamma, \Delta, \Pi, \Psi$ are sequences of types ($\Pi, \Psi \neq \Lambda$), A, B, C are types.

Definition

A sequent $\Pi \to A$ is derivable if it can be obtained from axiom sequents using rules. This is denoted as $\mathrm{L} \vdash \Pi \to A$.

Examples

- $\mathrm{L} \vdash np, (np\backslash s)/np, np \to s$:

$$\dfrac{\dfrac{s \to s \quad np \to np}{np, np\backslash s \to s}\ (\backslash \to) \qquad np \to np}{np, (np\backslash s)/np, np \to s}\ (/ \to)$$

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- $\mathrm{L} \vdash p \cdot (p\backslash q) \to q$:

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It generates the language of the strings $a_1 \dots a_n$ over Σ such that:

$$\begin{array}{cccc} & a_1 \quad \dots \quad a_n & \\ & \triangleright \qquad\quad\; \triangleright & \\ \mathrm{L} \vdash & T_1, \dots, T_n & \to S \end{array}$$

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- Distinguished type: s (sentence).
- This grammar accepts strings like *Olaf sleeps*, *Olaf loves Astrid* etc.

$$\begin{array}{ccccc} & Olaf & loves & Astrid & \\ & \triangleright & \triangleright & \triangleright & \\ \mathrm{L} \vdash & np, & (np\backslash s)/np, & np & \to s \end{array}$$

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- Hypergraph Lambek calculus HL and hypergraph Lambek grammars.

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- Hypergraph Lambek grammars are able to generate more languages than hyperedge replacement grammars (the language of all graphs, languages with non-linear growth of the number of edges etc.).
- What class of languages do hypergraph Lambek grammars generate?

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- A hypergraph consists of nodes V and of hyperedges E. A function $att : E \to V^*$ attaches hyperedges to nodes; a function $lab : E \to C$ labels hyperedges; $ext \in V^*$ are external nodes.

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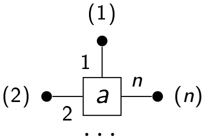
  4. The result is denoted as $G[e_0/H]$.
- $a^\bullet$ is a hypergraph with one a-labeled hyperedge attached to n distinct nodes, which are all external in the same order as attachment nodes:



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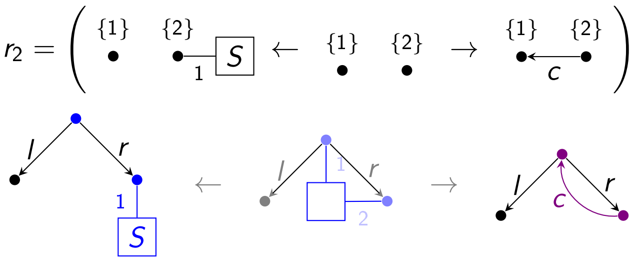
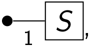
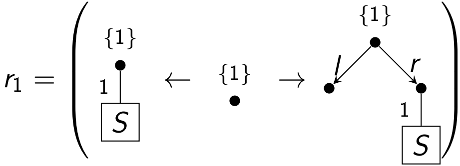
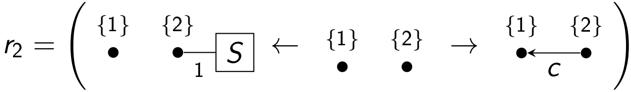


DPO grammar — Example

Let the start hypergraph be , and let the productions be as follows:





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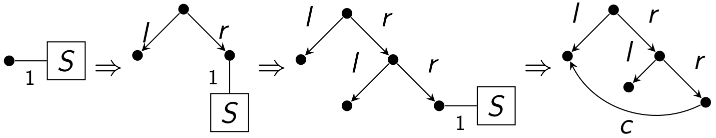

Results of this work

Definition

Given a DPO hypergraph grammar with a start hypergraph S and a set of productions P, let $L_c(HGr)$ consist of all hypergraphs $H \in L(HGr)$ such that there exists a derivation $S \overset{*}{\Rightarrow}_P H$ with no more than $c \cdot |E_H|$ steps.

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If HGr is a DPO grammar and $1 \leq c \in \mathbb{N}$, then the language $L_c(HGr)$ can be generated by a hypergraph Lambek grammar.

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Claim: each language generated by a hypergraph Lambek grammar is of the form $L_c(HGr)$ for some DPO grammar HGr.

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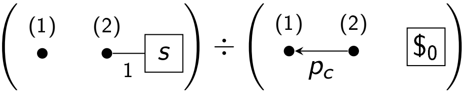
Example

Let s, p_c be primitive types.

$\mathrm{DPO}(r_2) = \times$



Hypergraph Lambek Calculus — Sequents

Definition

A sequent is a structure of the form $H \to A$ where H is a hypergraph labeled by types and A is a type.

Hypergraph Lambek Calculus — Axiom and Rules

Axiom: $A^\bullet \to A$ (A is a type).

$$\frac{H[e/N^\bullet] \to A \quad H_1 \to lab_D(d_1) \quad \dots \quad H_k \to lab_D(d_k)}{H\left[e/D[e^\$_D/(N \div D)^\bullet][d_1/H_1]\dots[d_k/H_k]\right] \to A}\ (\div \to)$$

$$\frac{D[e^\$_D/F] \to N}{F \to N \div D}\ (\to \div)$$

$$\frac{H_1 \to lab_M(m_1) \quad \dots \quad H_l \to lab_M(m_l)}{M[m_1/H_1]\dots[m_l/H_l] \to \times(M)}\ (\to \times)$$

$$\frac{H[e/M] \to A}{H[e/(\times(M))^\bullet] \to A}\ (\times \to)$$

Here $e \in E_H$; $E_D = \{e^\$_D, d_1, \dots, d_k\}$; $E_M = \{m_1, \dots, m_l\}$.

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This grammar generates the language of hypergraphs G over Σ, for which a function $f_G : E_G \to Tp$ exists such that

1. $lab_G(e) \triangleright f_G(e)$ for all hyperedges $e \in E_G$;
2. If $f_G(G)$ is the result of relabeling of G using f_G, then

$$\mathrm{HL} \vdash f_G(G) \to S.$$

From DPO to Hypergraph Lambek Grammars

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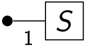
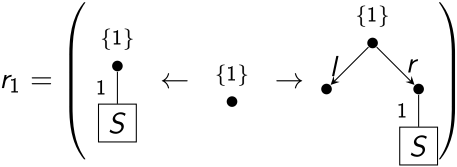
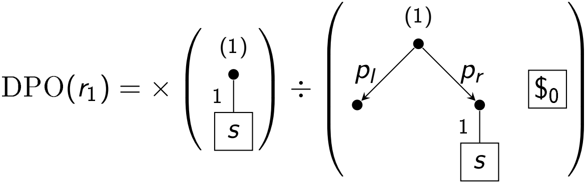
- Each rule application leaves a "trace", namely, a new type appears after any rule application.
- To prove the theorem we encode DPO rules by types of the hypergraph Lambek calculus.

Example of a convertion

Consider the DPO grammar with the start hypergraph 

and with the productions r_1, r_2. We introduce new primitive types p_l, p_r, p_c, s.





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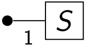
Consider the DPO grammar with the start hypergraph 

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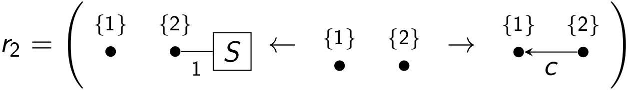
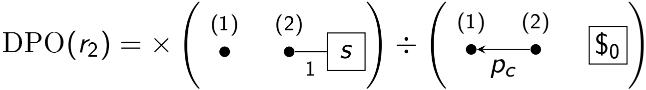




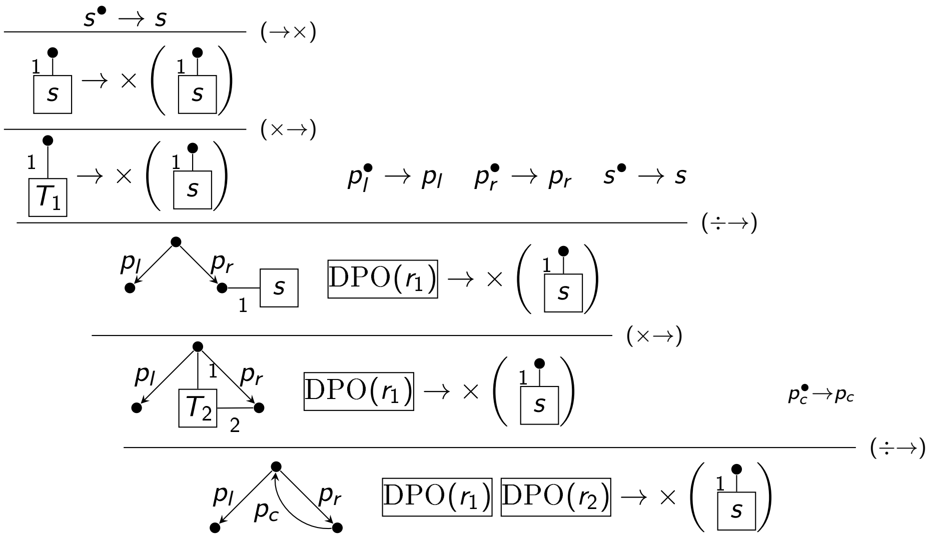
Example of a convertion

If we want to generate $L_2(HGr)$, then we construct the following grammar:

1. Alphabet: l, r, c.
2. A correspondence:
   - $l \triangleright p_l$, $r \triangleright p_r$, $c \triangleright p_c$;
   - $l \triangleright \times\left((1)\ \bullet \xrightarrow{p_l} \bullet\ (2) \quad \boxed{\mathrm{DPO}(r_i)} \right)$, $r \triangleright \times\left((1)\ \bullet \xrightarrow{p_r} \bullet\ (2) \quad \boxed{\mathrm{DPO}(r_i)} \right)$,
     $c \triangleright \times\left((1)\ \bullet \xrightarrow{p_c} \bullet\ (2) \quad \boxed{\mathrm{DPO}(r_i)} \right)$;
   - $l \triangleright \times\left((1)\ \bullet \xrightarrow{p_l} \bullet\ (2) \quad \boxed{\mathrm{DPO}(r_i)} \ \boxed{\mathrm{DPO}(r_j)} \right)$,
     $r \triangleright \times\left((1)\ \bullet \xrightarrow{p_r} \bullet\ (2) \quad \boxed{\mathrm{DPO}(r_i)} \ \boxed{\mathrm{DPO}(r_j)} \right)$,
     $c \triangleright \times\left((1)\ \bullet \xrightarrow{p_c} \bullet\ (2) \quad \boxed{\mathrm{DPO}(r_i)} \ \boxed{\mathrm{DPO}(r_j)} \right)$.

   Here $i, j = 1, 2, 3$.
3. A distinguished type: $\times\left(\bullet \underset{1}{\text{—}} \boxed{s} \right)$.

Example of a derivation



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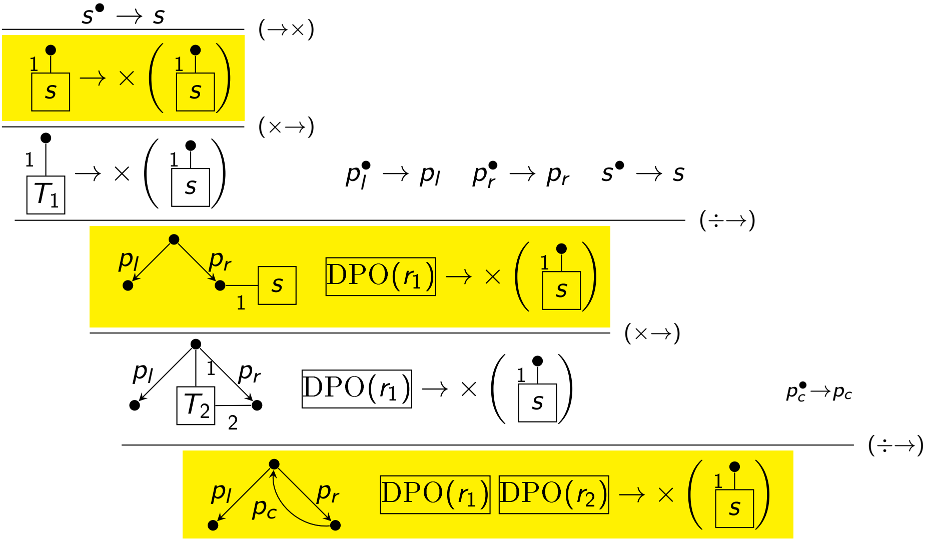

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Theorem

If HGr is a DPO grammar and $1 \leq c \in \mathbb{N}$*, then the language* $L_c(HGr)$ *can be generated by a hypergraph Lambek grammar.*

- The limitation is essential: DPO grammars are universal while HL-grammars are NP-complete.

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- Q: How to generalize the hypergraph Lambek calculus in order to overcome this limitation?
- A: To allow one unlimitedly copying types of the form $\mathrm{DPO}(r)$ in left-hand sides of sequents.

Hypergraph Conjunctive Kleene Star

- Palka 2007, Kuznetsov 2021, . . . : the Lambek calculus enriched with Kleene star (aka action logic).
- It allows one to unlimitedly copy types in right-hand sides of sequents.

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Definition

The hypergraph Lambek calculus with conjunctive Kleene star ${}^{*}\mathrm{HL}_{\omega}$:

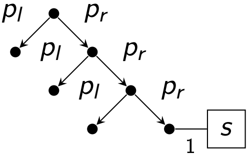
- Types are built using an additional constructor: if A is a type of rank 0, then $\overset{*}{\circ}A$ is also a type.
- We add the following rules:

$$\frac{(H \to \times(n \cdot A^{\bullet}))_{n=0}^{\infty}}{H \to \overset{*}{\circ}A}\ (\to^{*})_{\omega} \qquad\qquad \frac{G + n \cdot A^{\bullet} \to B}{G + \overset{*}{\circ}A^{\bullet} \to B}\ ({}^{*}\!\to),\ n \geq 0$$

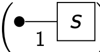
Here $G + H$ is a disjoint sum of hypergraphs, and $n \cdot H$ is H copied n times.

Hypergraph Conjunctive Kleene Star

- Palka 2007, Kuznetsov 2021, . . . : the Lambek calculus enriched with Kleene star (aka action logic).
- It allows one to unlimitedly copy types in right-hand sides of sequents.



$\boxed{\mathrm{DPO}(r_1)}\ \boxed{\mathrm{DPO}(r_1)}\ \boxed{\mathrm{DPO}(r_1)} \to \times\Big($



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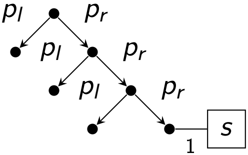
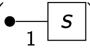


$\boxed{\overset{*}{O}(\mathrm{DPO}(r_1))} \to \times\Big($



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Results

Theorem (cut elimination)

If ${}^*\mathrm{HL}_\omega \vdash H \to A$ *and* ${}^*\mathrm{HL}_\omega \vdash G[e/A^\bullet] \to B$, *then* ${}^*\mathrm{HL}_\omega \vdash G[e/H] \to B$.

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The problem of whether a sequent is derivable in $^*\mathrm{HL}_\omega$ *is undecidable.*

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Thank you for attention!